Assignment # 1	Due on October 16th in class

1. For l^∞ defined as

$$l^\infty = \{(\xi_1, \xi_2, \cdots)\colon \sup_{k\in\mathbb{N}_{\geq 1}} |\xi_k| < \infty\},$$

define

$$d\colon l^\infty \times l^\infty \longrightarrow \mathbb{R}_{\geq 0},\ (x, y) \mapsto \sup_{k\in\mathbb{N}_{\geq 1}} |\xi_k - \eta_k|,$$

for all $x = (\xi_1, \xi_2, \cdots) \in l^\infty$ and $y = (\eta_1, \eta_2, \cdots) \in l^\infty$.

Prove that d is a metric on l^∞.

2. On $C[a, b]$, define

$$d\colon C[a, b] \times C[a, b] \longrightarrow \mathbb{R}_{\geq 0},\ (x, y) \mapsto \int_a^b |x(t) - y(t)|\,\mathrm{d}t$$

for all $x, y \in C[a, b]$.

Prove that this d is a metric on $C[a, b]$.

3. Let X be a set. Assume that d_1 and d_2 are two metrics on X. Define

$$d\colon X \times X \longrightarrow \mathbb{R}_{\geq 0},\ (x, y) \mapsto \max(d_1(x, y), d_2(x, y)).$$

Prove that the above defined d is a metric on X.